

Holographic creation of arbitrary potentials for Bose-Einstein condensates with digital micromirror devices.

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Abstract: In this work, I describe a method for creating holograms with a digital micromirror device. The purpose is to later use holograms to create arbitrary potentials for the potassium Bose-Einstein condensate experiment of the ICFO's Quantum Gases Group. The optical aberrations of the DMD and optical system used to create holograms are measured and corrected. I have been succesful in the creation of holograms in the Fourier plane.

I. INTRODUCTION

This work has been done in the Quantum Gasses group at the Institut de Ciències Fotòniques (ICFO). The group's main experiment is a Bose-Einstein condensate (BEC) of potassium atoms. In 2015 it became the first experiment that created a BEC in Spain.

It is of interest to trap the atoms of a BEC in arbitrary potentials. This can be done with light. Light fields induce an electric dipole moment in the atoms, that makes them sensible to electric fields, including the electric field of the light field itself. The resulting optical dipole potential is proportional to E^2 , meaning it is proportional to the intensity of the light field.

To create arbitrary light potentials, light should be shaped to the profile desired. This can be done with a Digital Micromirror Device (DMD). DMDs are arrays of micron sized mirrors capable of changing orientation between an *on* and an *off* state. DMDs can modulate light into binary patterns, sending parts of the incident beam to the *on* or *off* direction. These directions are defined by the blazed diffraction gratings created by the *on* mirrors and the *off* mirrors.

There are two main methods to generate potentials using the DMD. One option is to display a binary image on the DMD, and through direct imaging, use it to generate potentials for the atoms. Another option is to work in the Fourier plane to generate arbitrary light patterns, meaning to Fourier transform the light coming from the DMD using a lens. Compared to direct imaging, the Fourier plane is good for creating small and detailed potentials, whereas direct imaging is good for smooth and extended potentials[1].

Holography can be used to further control the light coming from the DMD before Fourier transforming it. The strategy to create arbitrary potentials will be to holographically create the inverse Fourier transform of the desired potential using the DMD, so it can be seen in the Fourier plane. Moreover, the holographic technique combined with working in the Fourier plane allows us to measure and correct the aberrations introduced by the DMD in order to create very precise potentials.

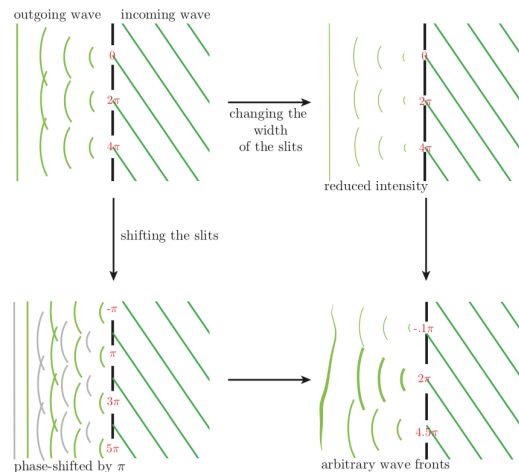


FIG. 1. Amplitude and phase modulation capabilities of a grating. Extracted from [1].

II. HOLOGRAPHY

Holography is the recording and reproduction of light fields. It allows us to control both the amplitude and phase of light beams. The method I used to create holograms is based on diffraction gratings. It is not the only way to create holograms, but it is the easiest to implement using a DMD.

Let us imagine a diffraction grating composed of equal spaces of reflective and non reflective material. If a planewave illuminates the grating, multiple diffraction beams will form. This beams are created by the constructive interference of the light coming from each reflective zone. In the 1st diffraction order, the phase difference between the light emitted by each reflective elements is 2π . If the angle of the illuminating the grating is adjusted such that the beam of the 1st order exits the grating perpendicularly, we have the situation shown in FIG. 1. If the whole grating is shifted, the wavefront of the outgoing beam will move forwards or backwards because the light reflected has a different phase. If we shift one reflective element, only light coming from that element will change phase. This way it is possible to locally mod-

ulate the phase of the beam. If the size of the reflective element is changed, the amount of intensity it reflects changes, this is the way to modulate intensity.

The mirrors of the DMD can be set *on* and *off* to create a grating that diffracts light as desired. A two dimensional homogeneous diffraction grating can be defined by a wave vector k_x and k_y by defining the phase of the grating p such that:

$$p \equiv k_x x + k_y y \mod 2\pi, \quad (1)$$

where x and y are the directions in the DMD plane and perpendicular to the optical axis, and then setting the mirrors *on* or *off* according to:

$$\text{Mirror state}(x, y) = \begin{cases} \text{on} & p \leq \pi \\ \text{off} & p > \pi \end{cases} \quad (2)$$

To shift the grating locally in order to introduce a phase $\phi(x, y)$ to the 1st diffraction order, the value of p can be redefined as:

$$p \equiv k_x x + k_y y + \phi(x, y) \mod 2\pi \quad (3)$$

The width of the reflective intervals should be adjusted according to the desired intensity profile $I(x, y)$, this is done by changing the conditions in equation 2 to a more general:

$$\text{Mirror state}(x, y) = \begin{cases} \text{on} & p \leq w(x, y) \\ \text{off} & p > w(x, y) \end{cases} \quad (4)$$

where w should equal π for the maximum intensity and 0 for 0 intensity. The intensity diffracted by a grating is proportional to the squared sine of the width of the reflective elements. Knowing this, a good definition for w is:

$$w(x, y) \equiv 2 \arcsin \sqrt{\frac{I(x, y)}{\max[I(x, y)]}} \in [0, \pi] \quad (5)$$

As I explained at the introduction, the holographic technique proposed consists on Fourier transforming the beam created by holography. An ideal thin lens can be used to optically obtain the Fourier transform. If an object is in the focal plane of an ideal thin lens, its Fourier transform appears in the other focal plane, now called *Fourier plane*[2].

In reality, when I set up the DMD with a grating to create an hologram with $\phi(x, y) = 0$, $\frac{I(x, y)}{\max[I(x, y)]} = 1 \forall x, y$, the created beam has $\phi(x, y) = \Delta\phi(x, y)$ and intensity $I(x, y) = I_m(x, y)$ where I_m is mostly due to the shape of the incident beam. The grating can only reduce intensity, meaning the intensity will be limited by the incident beam. The phase of the out-coming beam is due to aberrations introduced by the glass pane that covers the array of micromirrors.

Optical aberrations are the differences between the real behavior of an optical element and the behavior predicted

by paraxial optics. The protective glass pane differs from its ideal behavior due to the DMD manufacturing process. The glass pane is thermally glued to the structure of the DMD, in the process, tensions arise across the glass. This tensions locally change the refractive index, this means that, if a flat wavefront crosses the pane, the transmitted wavefront is no longer flat. $\Delta\phi(x, y)$ is the wavefront aberration due to crossing the glass pane two times (before and after being diffracted by the micromirrors).

In this section I have described how to do holograms by using the first diffraction order of a non-uniform grating. In the following sections I will describe what procedure I followed to measure and correct $\Delta\phi(x, y)$ and $I_m(x, y)$

III. MEASURING THE WAVEFRONT

A. First attempts

First I set up a Twymann-Green interferometer to get a qualitative idea of the wavefront aberration. I split a beam in two, one is reflected by the DMD and the other is not, then I make them interfere. From this I noticed that wavefront aberrations are larger than one wavelength. To get the actual values of $\Delta\phi(x, y)$ and $I_m(x, y)$ I tried a method based on the Hartmann-Shack sensor described in ref. [3]. I was not successful in correcting the aberrations from the measurements obtained. Due to the length constraints of this report, I will not comment on this method. Nonetheless, I was successful by using the strategy proposed in ref. [4] and [1], which I describe below.

B. Interference in the Fourier plane

A grating described by eq. 2 is shown at the DMD, which is located in one of the focal planes of a lens. The system is aligned such that the Fourier transform of the 1st diffraction order can be seen in the other focal plane. If instead of the whole DMD two patches of the same grating are shown, they will focus in the same point in the Fourier plane because they have the same spatial frequency. This will create an interference pattern in the Fourier plane. For example, a small patch of the grating is shown at the center of the DMD (x_0, y_0) , where x and y are the coordinates perpendicular to the optical axis (parallel to the surface of the DMD), and another patch is shown far from the center at (x_1, y_1) . The 1st diffraction order coming from each of this patches will be:

$$\begin{aligned} \text{Center patch: } & \sqrt{I_m(x_0, y_0)} e^{i[kz + \Delta\phi(x_0, y_0)]} \\ \text{Out of axis: } & \sqrt{I_m(x_1, y_1)} e^{i[kz + \Delta\phi(x_1, y_1)]}, \end{aligned} \quad (6)$$

where z is the coordinate in the optical axis, being $z = 0$ the location of the DMD, and k is the wave number $2\pi/\lambda$,

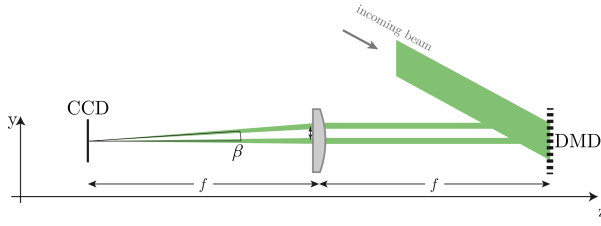


FIG. 2. Schematic of the setup to observe interference in the Fourier plane. Extracted from [1].

where λ is the wavelength. Going through the lens, the patch aligned with the optical axis will not change direction. The out of axis patch changes direction because it must go to the same point as the other beam in the Fourier plane. After the lens, and with respect to the focusing point, the out of axis beam will have an azimuthal angle α and latitude β :

$$\begin{aligned} \text{Center patch:} & \quad \sqrt{I_m(x_0, y_0)} e^{i[kz + \Delta\phi(x_0, y_0)]} \\ \text{Out of axis:} & \quad \sqrt{I_m(x_1, y_1)} e^{i[k(\sin\alpha x + \sin\beta y + z) + \Delta\phi(x_1, y_1)]}, \end{aligned} \quad (7)$$

where the paraxial approximation has been used. The interference $I_{\text{int.}}(x, y)$ seen in the Fourier plane can be found by adding the beams in $z = -2f$ and squaring the absolute value:

$$I_{\text{int.}}(x, y) \propto \cos[k(\sin\alpha x + \sin\beta y + \Delta\phi(x_1, y_1) - \Delta\phi(x_0, y_0))] \quad (8)$$

From this interference pattern I can measure the phase difference between two zones of the DMD. By choosing the center patch as the reference patch and scanning the rest of the DMD with a sample patch, I can build a phasemap relative to the reference patch. Phase is a relative value, so this phasemap is a measure of $\Delta\phi(x, y)$. The intensity profile $I_m(x, y)$ can also be measured from this interference pattern.

C. Optical setup and measurements

The optical system that I set up can be seen in FIG. 3. I used a 768.97 nm laser as a source of light. The laser is transmitted via a polarization maintaining fiber from the main experiment to my setup. Due to the length of the fiber (10 meters) and a non-optimal adjustment of the input polarization, the intensity of the laser oscillates slowly over time around a power of 0.6 mW. I used a power meter and an *Arduino* to log the changes in power.

The DMD used is the Texas Instruments DLP3000 mounted on a DLP LightCrafter evaluation module. This DMD features an array of 608×684 micromirrors in a diamond pattern. The diagonal length of these mirrors is $10.8 \mu\text{m}$ [5]. The DMD is positioned such that the on direction exits perpendicularly. The required incident

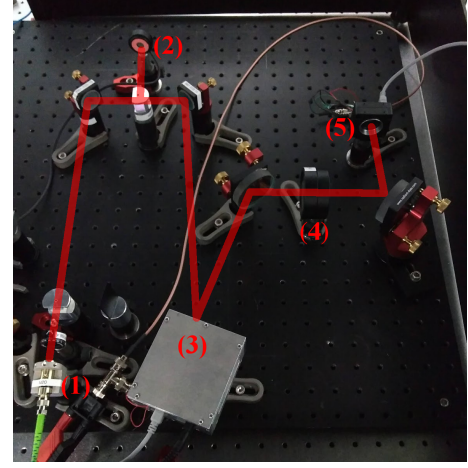


FIG. 3. Optical setup used to measure the wavefront and create holograms. (1) is the exit of the fiber, the collimator at the end enlarges the beam to about a waist of 5 mm, (2) is the sensor of a power meter, (3) is the DMD, (4) is the 200 mm focal length used to obtain the Fourier plane and (5) is the CCD sensor located at that plane.

angle so that the highest powered diffraction order exits perpendicular can be computed from the *blazing condition*, described in ref. [6]. The incident angle is $\theta_i \simeq 24^\circ$. Patterns can be uploaded via USB cable using a program provided by Texas Instruments.

Fourier transforming is done with a 200 mm focal length achromatic lens and the Fourier plane is recorded with a CCD sensor. Alignment of this lens has to be done carefully, as it can easily introduce defocusing aberrations and astigmatism.

To measure the interference patterns and later create holograms, I defined a grating with periodicity $100 \mu\text{m}$ with a direction that forms a 45° angle with the optical table (so it is aligned with the mirrors). I divided the surface of the DMD in $32 \times 18 = 576$ patches of 19 by 38 mirrors. As I explained before, I used the patch in the center as a reference and made it interfere with the other patches. FIG. 4 shows one of the combinations of sample patch and reference patch and the interference pattern they create.

In order to make the measurements, each time a pattern is shown at the DMD, it triggers both the CCD, capturing an image, and the *Arduino*, measuring the power of the incident beam. Unfortunately, the memory of the DMD can only hold 96 patterns and the program by the manufacturer makes you upload them manually in groups of 96. To optimize, I programmed my own implementation that allows to queue any number of patterns.

Once I had the 576 interference patterns I tried fitting a cosine with an Airy disk envelope. This method was slow and did not work with some images, so I used another strategy. If the grating of the sample patch is shifted, the interference pattern will also shift (principle explained in section II). If a series of images with varying shifts is recorded, the interference pattern can be reconstructed

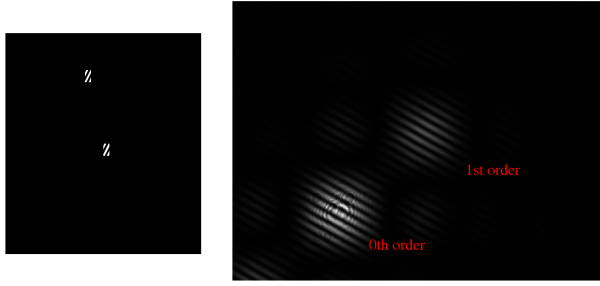


FIG. 4. On the left, one of the patterns shown at the DMD. On the right, the interference pattern created by the sample and reference patch.

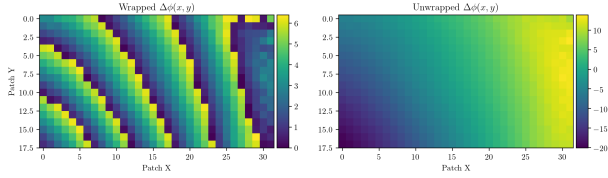


FIG. 5. Measure of the phase for each patch, at the top before unwrapping and at the bottom once the unwrapping has been applied. Each patch is formed of 19 mirrors.

by looking at one or a small group of pixels. Instead of fitting a function in a 2D space, the problem is reduced to fitting a cosine in 1D space. The downside of this strategy is that a minimum of three pictures are needed per patch.

The increase in quantity of images was not really a problem as I had already automated the process of taking any number of measures. I decided to take 10 images per patch, each one with an increasing shift of the sample patch's grating. To minimize the effects of intensity changes, the program uploads the 10 patterns of each patch and shows them all in a span 120 ms. In this time interval, power fluctuations of the laser beam are negligible. Once it is done, it uploads and then show the next 10.

By fitting cosines I obtain values for the phase map $\Delta\phi(x, y)$ and relative intensity map $I_m(x, y)/\max[I_m(x, y)]$ at the position of the patches. These measurements should be fitted to obtain maps with mirror resolution. When fitting phase data, one encounters the problem of *wrapped* phase maps. The phase measurement I found by fitting a cosine is always in the interval $[0, 2\pi)$, this means the phase map will not be smooth. *Phase unwrapping* consists in adding and subtracting multiples of 2π to phase data points in order to make the distribution as continuous as possible. I used an implementation in Python of the phase unwrapping algorithm proposed by [7]. FIG. 5 shows the phase map before and after unwrapping.

Once the unwrapping has been done, I fit a second degree polynomial to the phase map and a Gaussian

distribution to the intensity map to get $\Delta\phi(x, y)$ and $I_m(x, y)/\max[I_m(x, y)]$ with mirror resolution. These fits are shown in FIG. 6.

I have also fitted the first 15 Zernike polynomials to identify aberrations. Zernike polynomials are a base of orthogonal polynomials used to describe aberrations. The weight of each polynomial can be seen in FIG. 7. The largest contributions are Zernike polynomials with index 2 and 7. These are the polynomials associated with horizontal tilt and vertical coma. This is an indicator that the main source of aberrations is the misalignment of the optical system. Although it means I don't have a measurement of exclusively the DMD aberrations, the method proposed can correct for aberrations coming from the whole system.

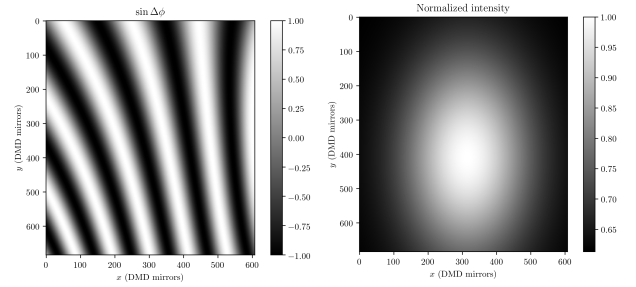


FIG. 6. Left, phase introduced by the optical system in each mirror, obtained from fitting to the phase of each patch. On the right, intensity profile of the first order of diffraction on the grating shown at the DMD.

IV. HOLOGRAMS

When creating holograms, $\Delta\phi(x, y)$ and $I_m(x, y)$ must be corrected or taken into account. Correcting phase is simple: if an hologram with phase $\phi_t(x, y)$ is wanted, the phase added to the grating should be:

$$\phi(x, y) = \phi_t(x, y) - \Delta\phi(x, y) \quad (9)$$

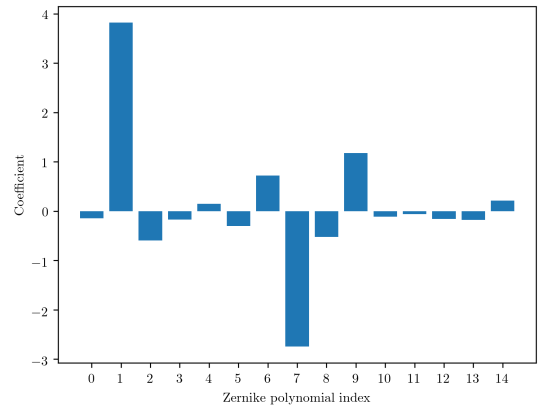


FIG. 7. Coefficient of each Zernike polynomial fitted to the phasemap.

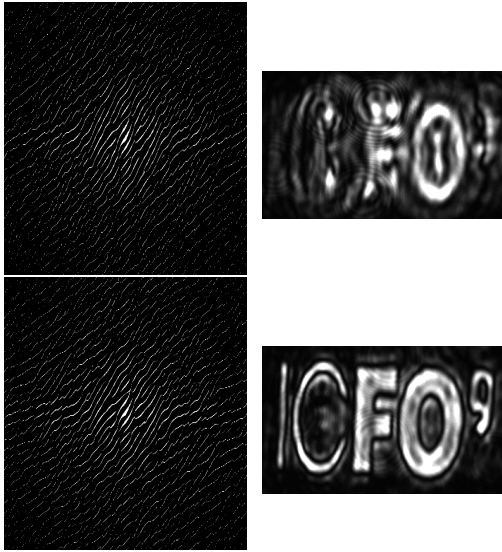


FIG. 8. Top left, diffraction grating without corrections, creates the hologram on the top right. Bottom left, corrected diffraction grating creates the hologram on bottom right.

Regarding to intensity, if a profile $I_t(x, y)$ is desired, the $I(x, y)$ used to define the local width of the slits should be:

$$I(x, y) = \frac{\max[I_m(x, y)]}{I_m(x, y)} \frac{I_t(x, y)}{\max[I_t(x, y)]} \quad (10)$$

To create a desired hologram with $I_U(x, y)$ and $\phi_U(x, y)$ in the Fourier plane, the inverse Fourier transform must be computed to obtain $I_t(x, y)$ and $\phi_t(x, y)$ to generate the grating.

To test the strategy, I added aberrations on purpose by misaligning the system. An example of modified grating and the hologram created can be seen in FIG. 8. The improvement due to corrections can also be seen with a hologram of a Gaussian beam with corrections and without corrections. A section of these beams can be found in FIG. 9.

V. CONCLUSIONS

- During this project I have learned about DMDs, holography and Fourier optics. I applied this ideas to set up

an optical setup to test the creation and correction of holograms.

- I automated the process of taking data, measuring aberrations and computing corrected holograms.
- I have been successful in holographically creating complex patterns while correcting the intensity and phase aberrations of the optical system.
- This is not an standard technique in the group, but one that will be implemented. It is not possible to put a CCD or photodiode inside the vacuum chamber where

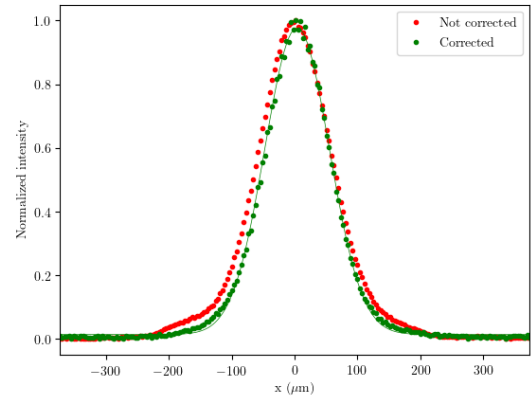


FIG. 9. Section of the holograms of Gaussian beams, corrected and not corrected.

the condensate is formed. As an alternative, the atoms forming the condensate could be used as a "sensor" to measure the aberrations introduced by the system, included the vacuum chamber window.

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[1] P. Zupancic, *Dynamic Holography and Beamshaping using Digital Micromirror Devices* (Master's thesis, Harvard University, 2013).
[2] S. Baek Oh G. Barbastathis, C. Sheppard, "Optics," <https://ocw.mit.edu/>, accessed: 2019-06-8.
[3] S. de Léséleuc, *Digital Micromirror Device* (Master's thesis, ETH Zurich, 2014).
[4] P. Zupancic *et al.*, "Ultra-precise holographic beam shaping for microscopic quantum control," *Opt. Express* **24**,

13881–13893 (2016).

[5] Texas Instruments, *DLP3000 DLP 0.3 WVGA Series 220 DMD datasheet (Rev. B)*.
[6] Christopher Palmer, *Diffraction Grating Handbook*, 6th ed. (Newport Corporation, 2005).
[7] M. Arevallilo Herráez *et al.*, "Fast two-dimensional phase-unwrapping algorithm based on sorting by reliability following a noncontinuous path," *Appl. Opt.* **41**, 7437–7444 (2002).